INFLUENCE OF GAPS IN THE BEAM ON SINGLE STREAM INSTABILITY

OF THE SPS BUNCHES

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1. **CONCLUSION**

A gap in the beam will not remove instability due to resistive wall or beam equipment interaction.

2. **INTRODUCTION**

W. Hardt has suggested to leave a gap in the SPS beam to avoid accumulation of electrons trapped in the proton beam. We want to make an estimate of the influence of this gap on single stream instabilities of the bunched SPS beam (resistive wall and/or beam equipment interaction).

The influence of the gap on the proton-electron two stream instability has been treated by H. Schönauer and B. Müller [1].

3. **RAPIDLY DECAYING WAKES, CLOSED LOOP INSTABILITY**

Assume that every bunch acts only on the subsequent one. Hence, neglect the influence of the wake fields on all other bunches.

In the absence of within bunch frequency spread, the equation of motion of bunch $e$ is:

$$\frac{d^2}{dt^2} x_e + \omega_e^2 x_e = 2 \omega_0 W_e \dot{X}_{e+1}$$

(1)

$$\omega_e \approx \omega_0 = \begin{cases} Q \omega_{\text{rev}} & \text{for transverse dipole motion} \\ Q_s \omega_{\text{rev}} & \text{for longitudinal dipole motion} \end{cases}$$

$W_e$ : Wake field experienced by bunch $e$ due to motion of bunch $e + 1$

The system (1) of coupled linear equations has been treated in reference 2. The characteristic equation which determines the $2N$ eigenfrequencies $\lambda$ of the system is easily obtained by inserting trial solutions...
of the type: 

\[ X_k = a_k e^{i \lambda t} \]

There is no exponential growth if all eigenvalues have a positive imaginary part.

Assuming \( \omega_k^2 - \lambda^2 \approx 2 \omega_k (\omega_k - \lambda) \), we find \( \lambda \) from:

\[
(\omega_1 - \lambda)(\omega_2 - \lambda) \cdots (\omega_M - \lambda) = W_1 \times W_2 \times \cdots \times W_M = (\bar{W})^M
\]

(3) Here, \( \bar{W} = \sqrt[M]{W_1 \times W_2 \times \cdots \times W_M} \)

is the geometric mean of the wake coefficients \( W \).

Given the bunch frequencies \( \omega_1, \omega_2, \ldots, \omega_M \), the stability behavior is entirely determined by the magnitude of \( \bar{W} \) (see reference 2).

To assess the influence of a gap, assume that:

\[
W_1 = \epsilon W \quad W_2 = W_3 = \ldots \quad W_M = W
\]

Here, \( \epsilon \) is a measure of the decay of the wakes during the passage of the gap. Conclude that the gap reduces \( \bar{W} \), equation (3), by a factor of \( \sqrt[M]{|\epsilon|} \). Hence compared to a machine with the same number of equally spaced bunches, the instability will be weaker by this factor (we assume \( M \gg 1 \)).

We determine \( \epsilon \) for the example of resistive wall wakes and wake fields induced in a low-Q cavity.

In the resistive wall case, wake fields decay as \( t^{-\frac{1}{2}} \) where \( t \) is the time from the passage of the bunch creating the wake.
Hence, neglecting wakes from previous revolutions and previous bunches, except for the neighbouring bunch

\[ \epsilon \approx \sqrt{\frac{t_b}{t_g}} \]

Here, \( t_b \) is a measure of the bunch distance and \( t_g \) a measure of the gap length.

Assuming a gap of 300 empty buckets in the SPS followed by \( M = 4320 \) filled buckets

\[ \epsilon \approx \left( \frac{1}{300} \right)^{\frac{1}{2}} \]

and the instability is reduced by a factor of

\[ \frac{M}{\sqrt{\epsilon}} \approx 1 - 1.2 \times 10^{-4} \]

In other words, the instability is practically unaffected (because this factor is close to unity).

In the case of a resonant cavity, the wakes decay like \( e^{-t/\tau} \)

\[ \tau = \frac{Q}{\omega_{\text{res}}} \]

is the decay constant of the resonator and \( Q \) its quality factor.

Hence,

\[ \epsilon = e^{-(t_g - t_b)/\tau} \]

\[ \frac{M}{\sqrt{\epsilon}} = e^{-(t_g - t_b)/(M\tau)} \]

Conclude that a sizable reduction of the instability can only be expected if the decay constant is extremely small, namely:

\[ \tau \ll \frac{t_g - t_b}{M} \approx \frac{t_g}{M} \]

Assuming, in accordance with the previous example, \( t_g = 1.5 \mu s \); \( M = 4320 \), we find a sizable reduction only if the decay time is of the order of 0.5 ns or shorter. This corresponds to a quality factor \( Q \) around only 10 in the GHz region (\( \omega_{\text{res}} \approx 2\pi \times 10^9 \) Hz) where one expects potentially dangerous
resonances of chamber sections in the SPS.

We conclude that the stability behavior is almost unaffected, even when the wakes decay to very small values during the passage of the gap.

4. RAPIDLY DECAYING WAKES, OPEN LOOP INSTABILITY

When the wake fields decay completely to zero during the gap, a somewhat different type of instability can still occur. This open loop instability was analyzed by M. Bell (3) and H.S. Hereward (4). They have shown that for M equal bunches the biggest amplitude increases for the first M e-folding times with approximately the same time constant, as in the closed loop instability.

It is easy to extend their results to include Landau damping and bunch to bunch frequency spread. The result is that for a chain of many bunches, the stability conditions too are very similar to those in the closed loop case.

In conclusion, even in the case where the wakes decay completely during the gap, an instability similar to the closed loop case can occur.

5. SLOWLY DECAYING WAKES

The other limiting case, where the wake fields decay slowly in comparison to the revolution frequency, is rather trivial. In this case, every bunch acts on every other in the same way and the gap has practically no effect.
6. References

[1] H. Schönhauer and B. Zotter - Contribution to the CERN Laboratory II Spring Study [to be published]


[3] M. Bell - Two programmes for studying the coupling caused by excitation of the resonator, between bunches in the CPS - CERN/MPS/DL 71-3