Crab Cavity RF Noise Studies

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June 22\textsuperscript{nd} 2015
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What would we like to know: emittance growth rate due to crab cavity RF noise
What can we control (through LLRF design) crab cavity noise spectrum
A formalism relating the two is necessary to set specifications and requirements for the crab cavity LLRF and to a certain extent evaluate the feasibility of these goals.

As a secondary issue, there is a trade-off between LLRF performance and crab cavity noise level. This formalism would allow us to fully evaluate this trade-off (for the time being, noise reduction seems to be the harder goal to reach).
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Methodology

A statistical approach is used to calculate the transverse emittance growth caused by momentum kicks created by the crab cavity RF noise. This is appropriate since:

- The transverse momentum kicks $\Delta p$ that will affect the beam are defined by a time-dependent function and follow a random (stochastic) process.
- The goal is to track the emittance growth of the whole bunch, it is necessary to obtain the ensemble average of the momentum kick effect over all particles in a bunch.

There are a few assumptions involved:

- The random process $\Delta p$ is assumed to have a mean of zero, and to be stationary.
- The emittance growth is small (no big changes in $\nu_b$, $\nu_s$, $\hat{\phi}$ etc).
- The chromaticity is not too high (less coupling with synchrotron motion)
Derivation: momentum kick to $\Delta x$

- We use a normalized phase space

\[
x = \frac{X}{\sqrt{\beta}} = \sqrt{2J} \cos \xi(s) \\
p = -\sqrt{2J} \sin \xi(s)
\]

and we track

\[
\epsilon = \frac{1}{2} (E[(x - E[x])^2] + E[(p - E[p])^2]) = E[(x - E[x])^2] = E[(p - E[p])^2] \approx E[x^2]
\]

since the tolerable emittance growth in the HiLumi LHC (4% per hour) is orders of magnitude slower than filamentation due to the betatron spread (tens of milliseconds).

- The normalized transverse position for a given particle at turn $n$ is given by

\[
x_n = \dot{x} \left[ \cos (2\pi \nu_b n) \cos \theta - \sin (2\pi \nu_b n) \sin \theta \right] + \sum_{k=0}^{n} \Delta p(kT_{rev}) \sin (2\pi \nu_b(n - k))
\]

so we can relate the momentum kicks due to the crab cavity noise with the transverse displacement.
Derivation: crab cavity noise to momentum kick

- The change in divergence due to the crab cavity noise momentum kick on a particle is given by
  \[ \Delta X' = \frac{e\Delta V}{E_b} \]
  where \( V = V_o(1 + \Delta A) \sin(\phi_n + \Delta \phi_n) \)
  is the crab cavity voltage \((\phi_n \approx 0)\).

- Then, the normalized momentum kick due to phase noise alone and received by a single particle is
  \[ \Delta p_{\phi,n} = \sqrt{\beta_{CC}} \frac{eV_o}{E_b} \cos(\hat{\phi} \cos (2\pi \nu_s n + \psi)) \Delta \phi_n \]
  whereas the normalized momentum kick due to amplitude noise alone is
  \[ \Delta p_{A,n} = \sqrt{\beta_{CC}} \frac{eV_o}{E_b} \sin(\hat{\phi} \cos (2\pi \nu_s n + \psi)) \Delta A_n \]

- Notice the sine dependence of the phase noise kick on the synchrotron motion
  and the cosine dependence of the amplitude noise kick.
Emittance growth due to RF noise

Putting all these pieces together with a couple of notepads, a few pencils, and lots of coffee, an expression has been derived relating the crab cavity noise power spectral density with the emittance growth rate

**Phase Noise**

\[
\frac{d\epsilon_x}{dt} = \beta_{cc} \left( \frac{eV_0 f_{\text{rev}}}{2E_b} \right)^2 \left\{ e^{-\sigma_{\phi}^2} \left[ I_0 \left[ \sigma_{\phi}^2 \right] + 2 \sum_{l=1}^{\infty} l_{2l} \left[ \sigma_{\phi}^2 \right] \right] \right\} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} S_{\Delta \phi} \left[ (k \pm \nu_b)f_{\text{rev}} \right] \rho (\nu_b) d\nu_b
\]

**Amplitude Noise**

\[
\frac{d\epsilon_x}{dt} = \beta_{cc} \left( \frac{eV_0 f_{\text{rev}}}{2E_b} \right)^2 \left\{ e^{-\sigma_{\phi}^2} \sum_{l=0}^{\infty} l_{2l+1} \left[ \sigma_{\phi}^2 \right] \right\} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} S_{\Delta A} \left[ (k \pm \nu_b \pm \nu_s)f_{\text{rev}} \right] \rho (\nu_b) d\nu_b
\]

Notice that there are only small differences between the two cases.

- \( S_{\Delta A}(f) \) is significantly lower than \( S_{\Delta \phi}(f) \) in most accelerators, and is not considered in previous work on this subject.
- This is NOT the case for the LHC. The transverse damper reduces \( S_{\Delta \phi}(f) \) but not \( S_{\Delta A}(f) \). Additionally, the bunch length is much longer.
- \( S_{\Delta \phi}(f) \) is sampled at \( \nu_b \) whereas \( S_{\Delta A}(f) \) at \( \nu_b \pm \nu_s \)

It is now possible to determine the expected growth rate with an estimate of the crab cavity noise power spectral density.
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What does the damper see?

- The ADT samples the average displacement of each bunch. As a result, it will not register the displacement due to amplitude noise.
- The ADT action will reduce the effect of the crab cavity PHASE noise though.

Reduction of phase noise kick with increased synchrotron oscillation amplitude.

Increase of amplitude noise kick with increased synchrotron oscillation amplitude.
**Damper correction**

- The damper will lead to a reduction of phase noise given by

\[
R_d(f) = \int_{\infty}^{\infty} \left\{ 1 - \frac{e^{-\sigma^2 fH}}{C(\sigma f)} \frac{G^2|H_{BTF}(f)|^2 + jG(H_{BTF}(f) - \bar{H}_{BTF}(f))}{1 + jG|H_{BTF}(f)|^2} \right\} \rho(f) df
\]

where \(H_{BTF}(f)\) is the beam transfer function, \(G\) the damper gain, \(\rho(f)\) the betatron distribution, and \(C(\sigma f) = e^{-\sigma f^2} \left[ I_0 \left[ \sigma f^2 \right] + 2 \sum_{l=1}^{\infty} I_{2l} \left[ \sigma f^2 \right] \right] \)

Noise PSD reduction versus tune for a gaussian tune distribution \((\sigma_\nu = 0.003)\).

- The implication here is that the damper will not only change the growth rate, but will differentiate the noise level experienced by parts of the bunch.
Damper correction

- The damper correction is a function of the tune distribution, the bunch length, and the damper gain
- Define $\alpha = \frac{G}{4\pi \sigma_{\nu b}}$
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Growth rate dependence on noise PSD

- With Kevin Li’s help, we adjusted HEADTAIL to test this formalism.
- The growth rate increases linearly with noise PSD as expected.
- Notice that this is a statistical process. We are interested in the average growth rate. (errorbars represent $\sigma$ over many trials)

Simulation results with different phase noise PSD (solid lines). Theoretically estimated trend line (dashed lines).

Transverse emittance growth rate with noise PSD at $\bar{\nu}_b$. 
The growth rate follows the predicted dependence on bunch length very well.

For the design bunch length (7.55 cm, $\sigma_\phi = 0.6325$ rad), the phase noise bunch length factor is 2.6 times higher (0.723 vs. 0.277) than amplitude noise.

Growth rate dependence on bunch length, phase noise.

Growth rate dependence on bunch length, amplitude noise.
Growth rate reduction from the damper (phase noise)

Simulations were performed to check the damper correction as a function of bunch length, damper gain, and tune distribution (not shown), with very encouraging results.

![Diagram showing correction factor as a function of α for ρ_{sim}. Assumed HL-LHC Operation Point G = 0.2, σ_{νb} = 3e − 3](image-url)

Damper correction factor as a function of α for ρ_{sim}. Assumed HL-LHC Operation Point G = 0.2, σ_{νb} = 3e − 3
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Emittance growth due to phase noise

Estimation with LHC main cavity measurements

- First, we estimate the expected emittance growth rate using the measured power spectral density of the LHC main cavities (figure).

- The transverse emittance would more than double over an hour with this power spectral density! \( V_c = 3.0 \text{ MV}, \epsilon_n = 2.5 \text{ microns}, \beta_{cc} = 4000 \text{ m} \)
  - Including the \( \approx 12 \) reduction through the action of the transverse damper.

- So, what do we do? Clever RF FB techniques are required combined with a reduction of the RF FB bandwidth.
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SPS Tests

In the end, a combination of LLRF parameter optimization and component improvement will be necessary.

As a result, measurements are essential to validate our models and decide on the optimal strategy. The SPS test bench will be valuable.

- Emittance growth in the SPS is dominated by other factors
- We faced a similar issue in the case of longitudinal emittance growth due to the main LHC RF system (growth dominated by IBS)
- Solution: artificially injected noise until we saw a result in the emittance growth rate
- Renovated SPS damper will allow detailed studies on the effect of the damper
Conclusions

- Future steps: look at distribution change (not just emittance)
- Conduct two-beam simulations to estimate possible reduction of phase noise effect due to beam-beam (with ABP)
- A wideband transverse damper (∼ 2 GHz) would help reduce the amplitude noise effects and thus relax the LLRF requirements
- Developed a formalism estimating transverse emittance growth due to RF noise
  - Early estimates are a bit alarming
  - Clever RF FB techniques will be necessary to achieve a 5% transverse emittance growth rate (∼ IBS)
  - Detailed studies on the way

Thank you for your attention!